The Uniaxial Compressive Strength Of Coal: Should It Be Used To Design Pillars?

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ABSTRACT

The Bureau of Mines has recently completed a comprehensive study of coal strength. More than 4000 individual test results from over 60 seams were extracted from the literature and combined in the most complete data base of the uniaxial compressive strength of coal ever assembled. In addition, more than 100 case studies of in-mine pillar performance were available in the Analysis of Retreat Mining Pillar Stability (ARMPS) data base.

Statistical analysis of this wealth of data has yielded valuable results. The data shows clearly that the "size effect" is related to coal structure. The widely-used Gaddy formula, which predicts a significant strength reduction as the specimen size is increased, was found to apply only to "blocky" coals. For friable coals, the size effect was much less pronounced or even non-existent.

Case histories of failed pillars are the best available data on in situ coal strength. This study found no correlation between the ARMPS stability factor of failed pillars and coal specimen strength. Pillar design was much more reliable when a uniform coal strength was used in all case histories.

BACKGROUND

The uniaxial compressive strength of coal was one of the very first issues addressed by early rock mechanics researchers. Bunting (1911) observed that "to mine without adequate pillar support will result, sooner or later, in a squeeze; the inherent effects of which are crushing of the pillars, the caving of the roof, and the heaving of the bottom." By testing anthracite specimens of various sizes and shapes in the laboratory, Bunting and his collaborators hoped to aid mine operators in "establishing the width of chambers and pillars." They soon found that "the crushing strength of small cubes is greater than that for large cubes; and, with a constant base area, the crushing strength becomes less as the height increases" (Daniels and Moore, 1907). Bunting apparently concluded that these two issues, the "size effect" and the "shape effect," prevented the direct use of laboratory strength results in design. His design equation was the first U.S. empirical coal pillar strength formula:

$$S_p = S_1 \, [0.70+0.30(w/h)] \qquad (1)$$

Where: S_p = Pillar strength
S_1 = Coal strength parameter
w = pillar width
h = pillar height

Bunting used the laboratory results to determine the <u>shape</u> of the formula (figure 1). The coal strength parameter was determined from analysis of in situ pillar failure ("actual squeezes" in figure 1). For anthracite pillars, it was set at 7 Mpa (1000 psi).

The basic approach employed by Bunting and his colleagues remained the state-of-the-art for much of this century. For example, Zern presented the following equation (2) in the 1928 edition of the Coal Miners Pocketbook :

$$S_p = S_1 \, (w/h)^{0.5} \qquad (2)$$

Zern's suggested value of the coal strength parameter is 4.8-7 MPa (700-1000 psi).

More than twenty years later, Gaddy (1956) attempted to provide the link between laboratory specimens and field strength. He attacked the size effect by testing coal cubes of various sizes from five seams. Gaddy concluded that the strength decrease with increasing specimen size could be expressed as:

$$k = S_c \, (d)^{-0.5} \qquad (3)$$

Where: k = Gaddy constant = estimated strength of a 2.5 cm (1 in.) cube
d = specimen dimension (in)

His work led to the widely used Holland-Gaddy pillar strength formula (Holland and Gaddy, 1956):

$$S_p = k \, (w)^{0.5}/h \qquad (4)$$

The Holland-Gaddy formula appears to have been the first in the U.S. to employ a seam-specific strength parameter determined from laboratory testing.

In situ testing of full-scale pillars in South Africa during the 1960's resulted in the concept of a "critical" specimen size beyond which the strength is constant (Bieniawski, 1968). The Bieniawski pillar strength formula (equation 5) employed this concept:

$$S_p = S_1 \, [0.64+0.36(w/h)] \qquad (5)$$

Where: S_1 = In situ coal strength

Following Hustrulid (1976), Bieniawski recommended that the in situ strength be determined from laboratory tests, and that the Gaddy formula be used to reduce the strength to that of a 1-m (36 in) critical-sized specimen (Bieniawski, 1984).

Others proposed versions of the Holland-Gaddy and Obert-Duvall (Bauschinger) formulas that employed the in situ strength parameter (Bieniawski, 1984). It may be noted that the in situ coal strength in equation (5) is functionally equivalent to the "coal strength parameter" in equations (1) and (2).

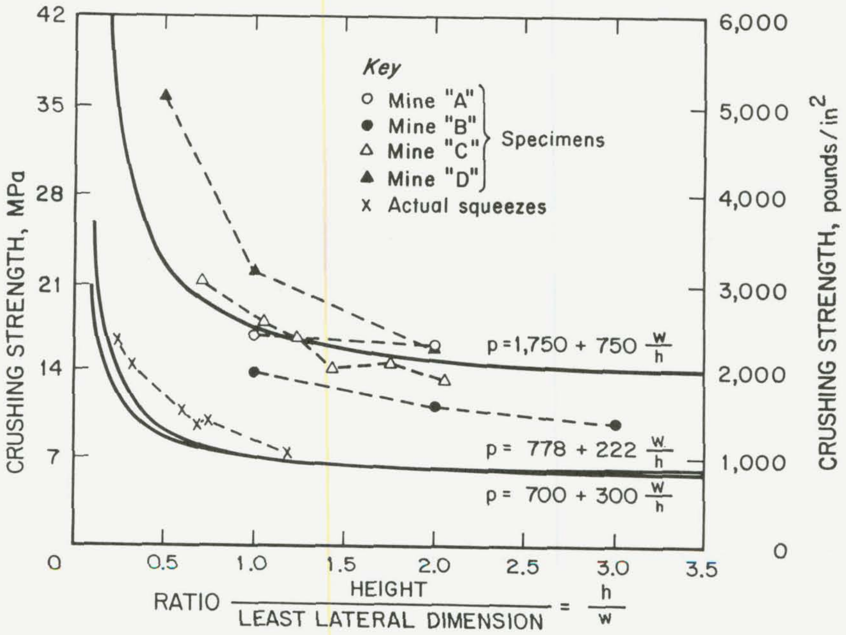
Despite the fact that textbooks have considered laboratory testing an integral part of pillar design for nearly 30 years, it has remained controversial. One reason is that coal remains notoriously difficult to test. Coal contains many types of discontinuities, including micro-fractures, cleats, bedding planes, partings, shears, and small faults. Three sources of unreliability have been identified:

1) Material variability within a particular seam: Unrug et al. (1985) tested multiple layers of the Warfield and the Coalburg seams, and found that the strongest layers were six times stronger than the weakest in each seam. Newman and Hoelle (1993) reported similar results from the Harlan seam.

2) Variation in sampling, specimen preparation, and testing techniques: Townsend et al. (1977) found that small cylindrical specimens were typically 30% weaker than cubical specimens of the same cross-section area. Khair (1968) documented large effects due to platen friction.

3) Variation in size and shape effects between seams: Panek (1994) and Mrugula and Belesky (1989), among others, have speculated that Gaddy's size effect exponent of -0.5 may be the maximum, and not universally applicable. The shape effect has been the subject of numerous studies.

Some have held that these difficulties, and the resulting high variability in results, are enough to largely invalidate laboratory testing.

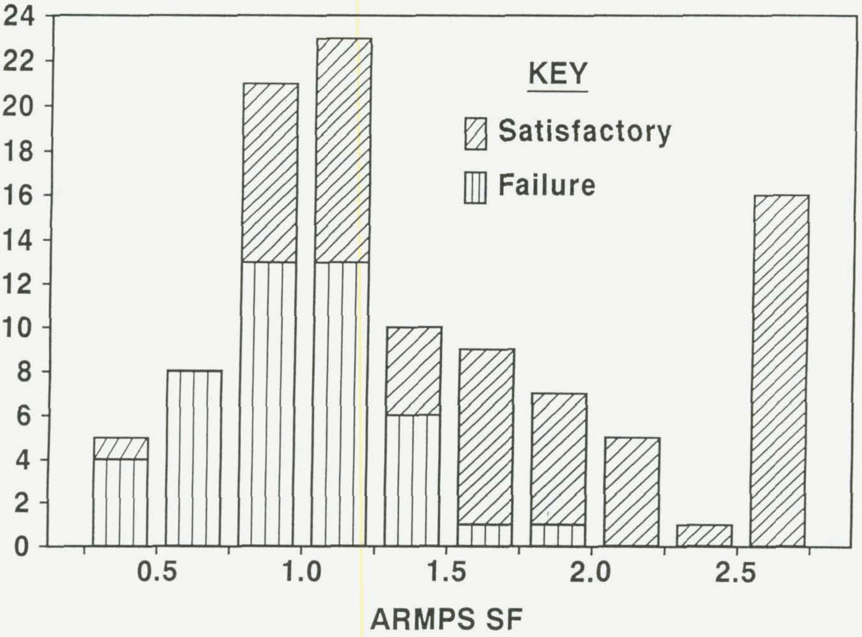


1. Data used to develop the first U.S. pillar strength formula (after Bunting, 1912).

Another school of researchers in South Africa, Australia, and the U.S. have argued that while the strength of laboratory-sized specimens varies widely, the in situ coal strength may fall within a narrow range (Salamon, 1991; Galvin, 1995; Mark, 1990). In each case their conclusions were based on analysis of in-mine pillar failures. Salamon and Munro (1967) originally analyzed 27 pillar collapses and 92 intact cases. Their formula, perhaps the most widely-used in the world, explained the data very well without reference to individual seam strengths. In 1991 Madden re-analyzed an updated version of their data set, and though he found some differences in strength between seams, he concluded again that the average strength could represent all seams. Galvin (1995) conducted a probabilistic analysis of 30 collapsed and stable bord and pillar workings from Queensland and NSW, Australia. He concluded that "pillar strength in the field is only marginally dependent on the seam strength once the w/h exceeds 2." In the U.S., Mark et al. (1995) presented data from 106 case histories, analyzed using the Analysis of Retreat Mining Pillar Stability (ARMPS). ARMPS estimates pillar strength using a slightly modified version of the Bieniawski formula, and the analyses assumed a uniform in situ coal strength. Mark et al. (1995) found that pillar failures occurred in 92% of cases when the ARMPS SF was less than 0.75, but only 8% of the cases when it greater than 1.5 (figure 2).

These writers have all determined that the value of the in situ coal strength falls between 5.4-7.4 MPa (780 and 1070 psi). The range is remarkably small, considering that it was determined from three data sets that span the globe. On the other hand, at least one South African seam has been shown by back-calculation to be significantly weaker than the average (Van der Merwe, 1993). In India, researchers concluded from back-analysis of 43 pillar case histories that coal strength should be considered in design (Sheorey et al., 1987).

Interest in the uniaxial compressive strength of coal has also waned over the past 15 years because researchers have devoted their energy to analytic pillar strength formulas and numerical models. These theories are developed from the principles of mechanics rather than curve-fitting to test data. The shift in emphasis has been related to the recent focus on pillar design for longwall mining. Longwalls employ pillars that are much more "squat" than the pillars traditionally used in room and pillar operations. Few compressive strength tests have ever been conducted where the specimen width-to-height ratio (w/h) exceeded 4, but longwall pillars often employ w/h of 10, 20 or even more.



2. The Analysis of Retreat Mining Pillar Stability (ARMPS) data base.

Obviously, the very concept of pillar failure takes on different meaning for squat pillars. The wide range of conflicting theories about the mechanics of squat pillars, and the substantial difficulties with obtaining field data to confirm or disprove any of them, have been described elsewhere (Mark and Iannacchione, 1992). On the other hand, Mark et al. (1994) have shown that longwall tailgate performance can be accurately predicted without reference to seam-specific coal strength. There is clearly overwhelming evidence, theoretical and empirical, that the uniaxial compressive strength is irrelevant to the strength of a squat pillar.

Longwall mines only account for 45% of the coal mined underground in the U.S., however. Much of the remainder comes from small room-and-pillar mines, usually operating at relatively shallow cover. These mines use lots of "slender" pillars, and traditional pillar failures still occur. The ARMPS data base contains more than 50 instances of pillar squeezes, bumps, or collapses that have taken place in recent years. About half of these occurred at depths of less than 150 m (500 ft) and involved pillars whose w/h ratio was less than 5. The failures occurred in a variety of seams. Since some seams appear blocky and strong, and others seem weak and extremely friable, it is reasonable to expect that these obvious structural differences might affect pillar strength. As figure 2 shows, successful and unsuccessful designs occur in approximately equal proportions in the ARMPS SF range of 0.75 to 1.5. Might seam-specific laboratory coal strength data explain some of this variability? That was the question this research was initiated to answer.

RESEARCH CONDUCTED

Despite the large volume of coal strength testing reported in the literature, it has never been compiled into a single data base. The Bureau therefore undertook the task. The Coal Strength Data Base now contains the results from more than 4000 individual uniaxial compressive strength tests, covering more than 60 seams, and obtained from more than 30 references. All the data has been entered into a spreadsheet, and is readily accessible for a wide variety of statistical studies.

Two types of data are included. For about 2300 tests, information was provided on single specimens. These data were entered individually, and then grouped by reference, seam, specimen geometry, and specimen size. Each group, or suite, of tests was placed on a separate page within the data base. A "summary line" containing the mean compressive strength and standard deviation for the suite was also generated. The summary lines were collected and placed in the summary table. The summary table also includes lines representing about 1700 tests that were reported in summary form in the original reference. The summary table contains information on about 380 suites of tests. The structure of the Coal Strength Data Base is illustrated in figure 3.

A single copy of the Coal Strength Data Base may be otained by sending three formatted, double-sided, high-density diskettes to: Timothy M. Barton, U.S. Department of Energy, Pittsburgh Research Center, Cochrans Mill Rd., P.O. Box 18070, Pittsburgh, PA 15236-0070. Please specify whether you prefer .xls, .wk3, or comma-separated values format.

A table of average U.S. Coalbed Strengths was derived from the summary statistics (Table 1). To minimize size and shape effects, this table uses only specimens whose w/h ratio is approximately 1.0, and whose least dimension is approximately 5-8 cm (2-3 inches). The average coalbed strength is calculated as the weighted mean of all the summary lines for a particular seam that meet these geometric criteria.

In addition to strength data, the Coal Strength Data Base also includes a variety of coal quality information for each seam tested. The most relevant of these is perhaps the Hardgrove Grindability Index (HGI), which is a measure of the relative grindability of coal. Larger HGI values imply easier grindability and greater friability. The HGI is almost universally required by utilities that purchase coal, so the information is readily accessible. Representative values of the rank, carbon content, volatile content, ash content and heating value are also included. Because the coal quality data were collected independently of the coal strength data, and from different sources, they are approximations for comparative purposes only.

During the past 6 years, coal samples measuring about 0.003 m^2 (0.1 cubic ft) have also been collected from 45 seams. These were classified using the following simple system:

Composition:

Bright (>90% bright coal)
Semi-Bright (60-90% bright coal)
Intermediate (40-60% bright coal)
Semi-Dull (60-90% dull coal)
Dull (>90% dull)

Structure:

Blocky (major cleat spacing>8 cm)
Semi-blocky (major cleat spacing 3-8 cm)
Friable (cleat spacing<3 cm (1 in))

Shearing: Yes or no

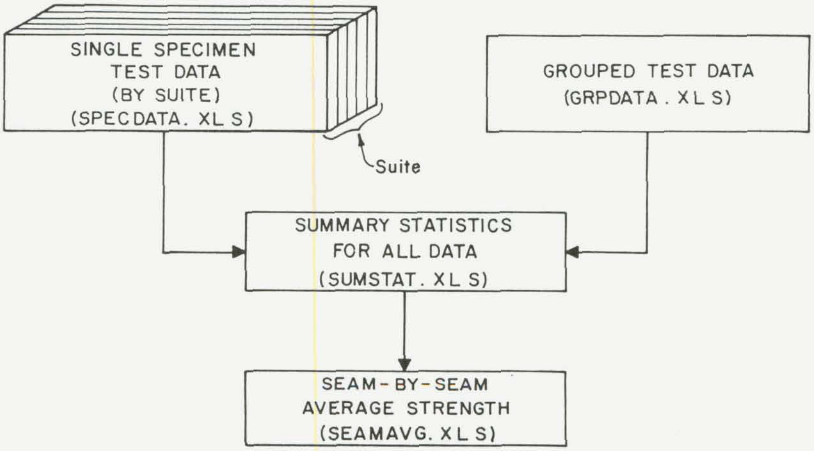


Figure 3. The structure of the Coal Strength Data Base.

The ARMPS data base contains the best available information on the in situ strength of U.S. coal pillars. ARMPS SF have been back-calculated for more than 100 case histories (figure 2), covering an extensive range of geologic conditions, extraction methods, depths of cover, and pillar geometries (Mark et al., 1995). Ground conditions in each case history have been categorized as being either satisfactory or unsatisfactory. Unsatisfactory conditions included:

- Pillar squeezes, with significant entry closure and loss of reserves;
- Sudden collapses of groups of pillars, usually accompanied by airblasts, and/or;
- Coal pillar bumps (violent failures of one or more pillars).

Table 1. Unconfined Compressive Strength (UCS) of U.S. Coal Seams (5-8 cm (2-3 inch) specimens).

Coalbed	Seam Average Strength MPA (psi)	Typical HGI	Number of Tests	Coalbed	Seam Average Strength MPA (psi)	Typical HGI	Number of Tests
Allen	10.8 (1570)	100	11	Kentucky No. 9	28.3 (4102)	54	46
Alma	27.7 (4024)	55	30	Lower Kittanning	14.6 (2117)	90	96
B Coalbed	25.1 (3633)	95	61	Marker	44.9 (6509)	47	24
Bakerstown	16.7 (2420)	64	12	Mary Lee	7.8 (1135)	76	10
Beckley	14.6 (2121)	101	30	No. 2 Gas	12.4 (1801)	52	50
Blind Canyon	38.9 (5646)	46	54	Pittsburgh	160 (4330)	56	160
Blue Creek	9.1 (1324)	81	10	Pocahontas No. 3	10.5 (1528)	110	85
Chilton	27.4 (3973)	50	6	Pocahontas No. 5	14.7 (2127)	100	4
Clintwood	19.2 (2783)	63	40	Pocahontas No. 4	19.9 (2892)	90	31
Coalburg	24.3 (3521)	45	124	Pond Creek	32.0 (4635)	39	13
D Coalbed	18.2 2632	46	10	Powellton	13.8 (2008)	58	13
Darby	20.9 (3907)	49	22	Redstone	20.2 (2932)	65	10
Douglas	15.9 (2300)	50	7	Sewell	16.5 (2386)	65	30
E Coalbed	24.2 (3514)	47	4	Sewickley	27.6 (4000)	60	72
Eagle	10.5 (1526)	59	10	Stockton	47.2 (6844)	45	10
Elkhorn No. 4	30.3 (4393)	42	24	Sunnyside	26.6 (3856)	50	48
Geneva	36.2 (5250)	48	3	Tiller	15.3 (2215)	54	12
Harlan	32.6 (4728)	44	88	Upper Banner	9.6 (1391)	84	30
Hazard No. 4	18.2 (2644)	43	67	Upper D	46.5 (6746)	50	36
Hernshaw	32.6 (4727)	47	10	Upper Freeport	10.3 (1493)	82	17
Herrin No. 6	24.7 (3576)	57	102	Upper Hiawatha	37.6 (5446)	46	20
Island Creek	32.6 (4734)	42	8	Upper Kittanning	10.5 (1519)	79	60
Jawbone	3.7 (539)	54	3	Warfield	22.7 (3295)	50	93
Kellioka	21.8 (3159)	44	49	Waynesburg	30.9 (4474)	54	15
Kentucky No. 11	25.5 (3693)	52	52	Welch	13.1 (1902)	95	6
Kentucky No. 12	15.6 (2268)	58	5	Winfrede	43.8 (6345)	45	10
Kentucky No. 13	26.8 (3890)	60	37	York	18.9 (2735)	54	60

RESULTS

Size Effect

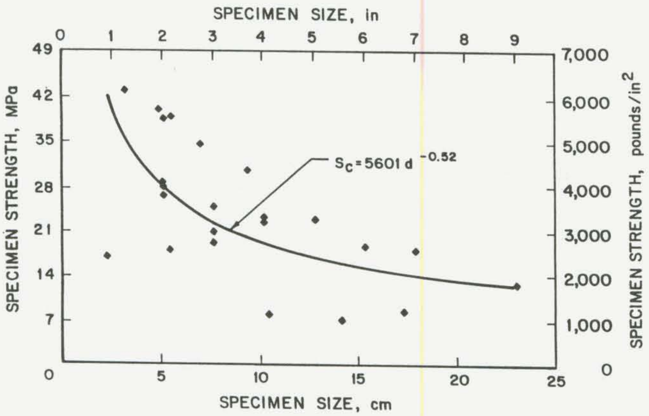
The Coal Strength Data Base contains information from 10 seams where a wide range of specimen sizes have been tested. Five of these were the seams originally tested by Gaddy.

To determine the size effect, only specimens with w/h of approximately 1:1 were used. Figures 4 and 5 show how power curves were fit to the data of the form:

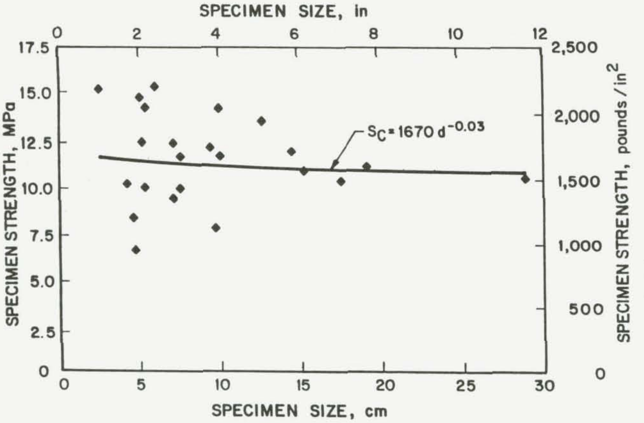
$$S_c = k\,(d)^{alpha} \qquad (6)$$

Where alpha = Size effect exponent.

The results are summarized in Table 2. Gaddy's alpha of -0.5 was found to apply to four seams, the Blind Canyon, Elkhorn, Pittsburgh, and Taggart-Marker. At the other extreme, the two Pocahontas seams displayed negligible size effect. The other four seams had intermediate size effects. The r^2 values indicate that the size effect typically explains about 50% of the variability in the test results, which is not bad considering all the potential sources of variation in these data.



4. Size effect in the blocky Pittsburgh seam.



5. Size effect in the friable Pocahontas No. 3 seam.

Table 2. Size Effect Exponents for 10 U.S. Coal Seams

Coalbed	Number of specimens	Number of references	Maximum Specimen Size (cm (in))	HGI	**Size Effect (alpha)**	k	r^2
Blind Canyon	126	2	29.5 (11.6)	46	**-0.54**	7045	0.90
Clintwood	88	1	18 (7)	63	**-0.31**	3686	0.93
Elkhorn	69	1	16.0 (6.3)	42	**-0.55**	7302	0.52
Harlan	129	2	18 (7)	44	**-0.29**	6491	0.31
Herrin No. 6	150	5	34.5 (13.6)	56	**-0.38**	4293	0.33
Taggart-Marker	60	1	18 (7)	47	**-0.45**	9837	0.99
Pittsburgh	272	7	22.9 (9.0)	55	**-0.52**	5601	0.48
Pocahontas No. 3	140	5	29.5 (11.6)	110	**-0.03**	1670	0.01
Pocahontas No. 4	74	1	18 (7)	100	**-0.13**	3238	0.58
Upper Banner	78	1	20.8 (8.2)	84	**-0.29**	1730	0.34

The explanation for the substantial range in size effect exponents is the different structure of the coalbeds. In a blocky coalbed, like the Pittsburgh (figure 4), a small sample will be largely free of cleats and fractures. As the specimen size increases, the density of cleating increases until it finally approaches in situ. In contrast, the fracture density of even a small sample of a friable seam like the Pocahontas No. 3 is nearly as great as in situ (figure 5). The following relationship between size effect and HGI was found (r^2=0.75):

$$\text{Alpha} = 0.75 - 0.0063\ \text{HGI} \qquad (7)$$

The implications of seam-specific size effects are quite important. It appears that the Gaddy equation underestimates the in situ strength of most seams, sometimes by a factor of 3 or more. Extremely costly and inefficient mining plans have certainly been the result.

Coal Structure

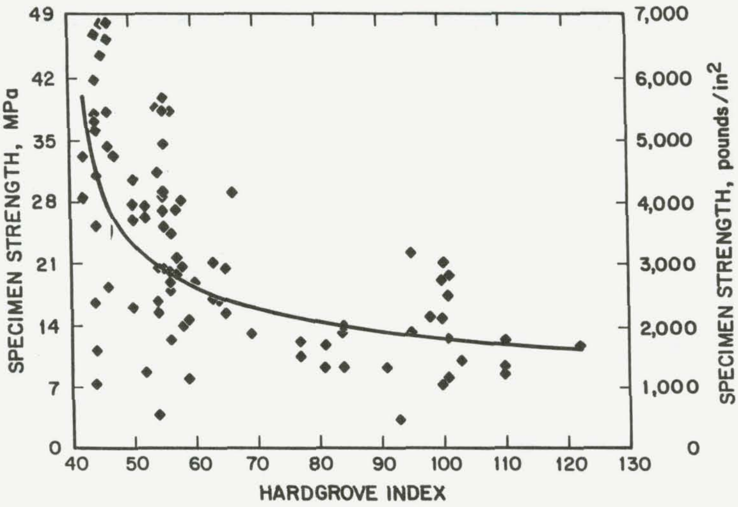
Several analyses explored the relationship between coal structure and specimen strength. Figure 6 shows U.S. Coalbed Strengths plotted against HGI. It shows that specimens from all seams with HGI>70 have strengths less than 20 MPa (3000 psi). These seams include all the medium- and low-volatile seams in the data base. Likewise, 85% of seams with HGI<50 have a strength exceeding 20 MPa (3000 psi). For the large number of seams in between these extremes, the HGI is a poor predictor of strength. Many of these intermediate HGI seams are high-volatile A in rank. The r^2 for the power curve fit to the entire data set is 0.33.

The second analysis compared the structure of the hand samples obtained from the mines with the HGI. In this case, every seam rated "blocky" had an HGI less than 60. The HGI of the "semi-blocky" seams was less than 80. "Friable" seams were found throughout the range of HGI.

Compressive strength and sample structure data were available for 26 seams. The specimen strength of all eight blocky seams exceeded 23 MPa (3500 psi), but so did that of four friable and one semi-blocky seams. Another 13 friable and semi-blocky seams were intermixed below 23 MPa.

ARMPS Case History Data Base

Coalbed specimen strength data were available for approximately 100 case histories in the ARMPS data base. The case histories are about evenly split between successes and failures. In figure 6, the ARMPS SF are plotted against coal strength. All ARMPS SF were calculated assuming the in situ strength was 6.2 MPa (900 psi). If pillar strength was related to specimen strength, low strength seams would be expected to fail at greater SF than high strength seams. Instead, no meaningful correlation between SF and coal strength is apparent in the data. The best discrimination is achieved at an ARMPS SF of 1.55, with a



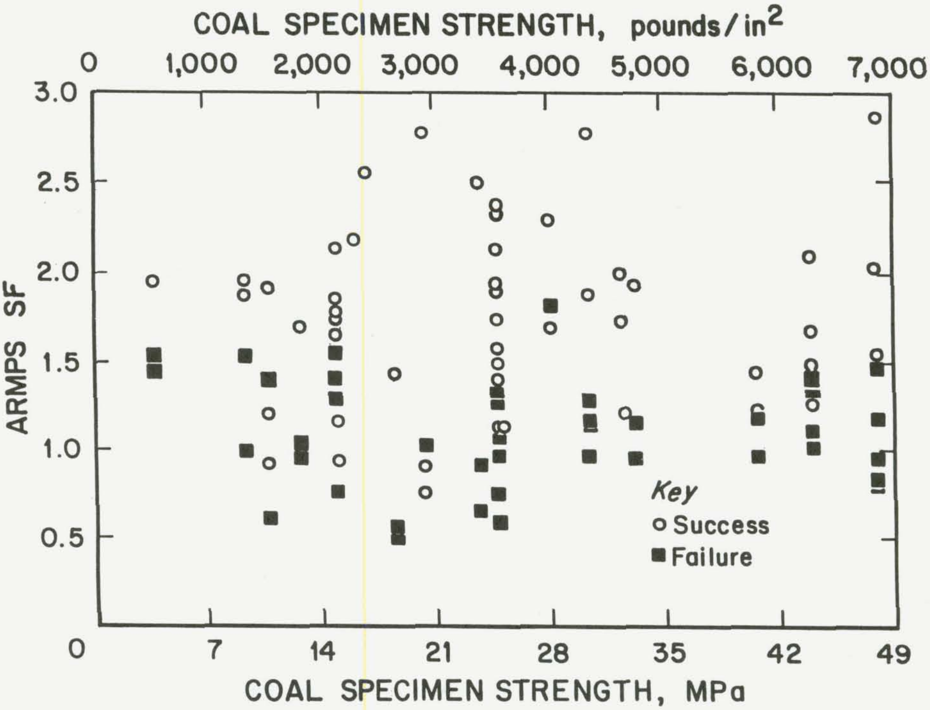
6. Specimen strength and HGI of U.S. coalbeds.

misclassification rate of 20%. Only 1 failure is included among the misclassifications, which is highly significant from a practical standpoint.

In a second analysis, the ARMPS SF were re-calculated using individual seam strengths instead of the uniform in situ strength. The seam strengths were divided by 4, as suggested by the Gaddy formula for a 6.5 cm (2.5 in) specimen, resulting in a mean SF that is about the same as in the first analysis.

The results are shown in figure 8. Now there is a strong correlation between specimen strength and SF, with "stronger" coals requiring higher SF to avoid failure. The best misclassification rate, at an SF of about 1.7, is 37%. Also, the misclassifications now include 10 failures. In other words, when seam-specific strengths are used, the SF becomes almost meaningless.

A third analysis applied seam-specific size-effect exponents to the coal strength data, using equation (6). The correlation between seam strength and SF was still apparent, as in figure 8. Although the misclassification rate improved to 33%, it was still 50% greater than in the uniform seam strength analysis.

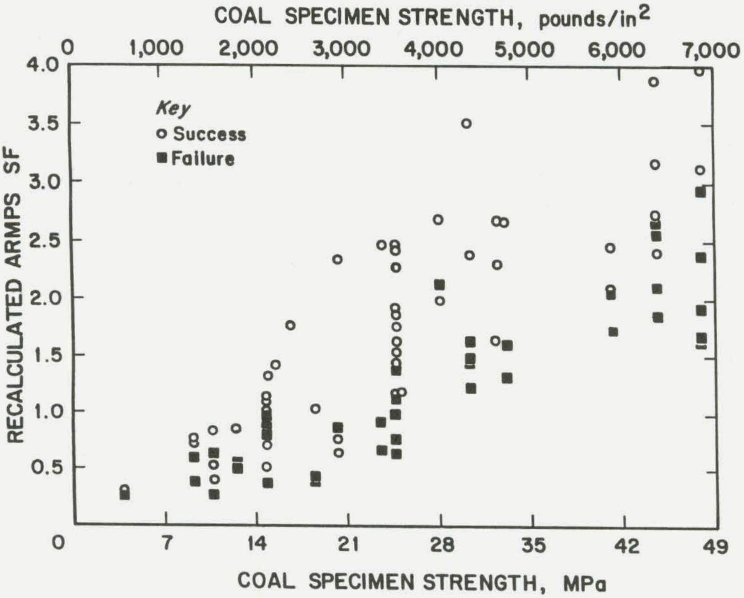


7. The Analysis of Retreat Mining Pillar Stability (ARMPS) Stability Factors compared with specimen strength data.

The data indicate that there is no meaningful correlation between the specimen strength and the in situ strength of U.S. coal seams. Knowledge of the specimen strength does not improve the accuracy of the design formula's prediction, it reduces it. A uniform coal strength provides a much more reliable prediction of pillar performance. Based on these results, laboratory test results are explicitly not recommended for use in ARMPS.

CONCLUSIONS

The results of this study cast doubt on many textbook assumptions about the value of coal strength testing. The data shows clearly that specimen strength, and the "size effect", are highly seam-specific, and related to coal structure. The widely-used Gaddy formula, which applies a uniform strength reduction for all seams as



8. Recalculated Analysis of Retreat Mining Pillar Stability (ARMPS) Stability Factors, using seam-specific coal strength data.

specimen size is increased, only applies to "blocky" coals with cleats spaced more than 8 cm (3 in.) apart. For friable coals, the size effect was much less pronounced or even non-existent.

Case histories of failed pillars are the best available data on in situ coal strength. This study found no correlation between the ARMPS stability factor of failed pillars and coal specimen strength in the ARMPS data base. In current ARMPS practice, pillars are designed assuming an in situ strength of 6.2 Mpa (900 psi) for all seams. When the specimen strength was used instead, the reliability of the ARMPS design method decreased substantially. Australian and South African studies have also found that pillar strength in the field is largely independent of specimen strength.

It should be noted that the coal strength tests were only matched with the seams in the case histories, and not with the individual mines. It is also possible that some of the case histories involved roof or floor failure rather than pillar failures. Using a different pillar strength formula might also have changed the results somewhat. But the data base is so large, and the trends so strong, that it is highly unlikely that the study is unrepresentative.

The most likely explanation for the study's results is that specimen and in situ strengths are determined by different parameters. Laboratory tests, particularly of blocky coals, require a significant amount of fracturing of intact coal. Pillars contain so many cleats and other discontinuities that their failure can occur almost entirely along pre-existing fractures. The laboratory tests measure a parameter, the intact coal strength, that is apparently irrelevant to the in situ strength.

The study did not prove that the in situ strength of all U.S. coals is uniform. It only showed that a uniform strength is a better approximation than one based on laboratory testing. There is still significant variability in the ARMPS SF range of 0.75 to 1.5. It may well be that features like bedding planes, partings, or weak coal layers do effect in situ strength. A rock-mass classification, like the one proposed by Kalamaras and Bieniawski (1993), may prove to be the best way of evaluating these effects. In the meantime, laboratory uniaxial coal strength test results should not be used for pillar design with ARMPS.

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