The Science of Empirical Design in Mining Rock Mechanics

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ABSTRACT

Many problems in rock mechanics are limited by our imperfect knowledge of the material properties and failure mechanics of rock masses. Mining problems are somewhat unique, however, in that plenty of real world experience is generally available and can be turned into valuable experimental data. Every pillar that is developed, or stope that is mined, represents a full-scale test of a rock mechanics design. By harvesting these data, and then using the appropriate statistical techniques to interpret them, mining engineers have developed powerful design techniques that are widely used around the world. Successful empirical methods are readily accepted because they are simple, transparent, practical, and firmly tethered to reality.

The author has been intimately associated with empirical design for his entire career. But where his past papers have described the *application* of individual techniques to specific problems, the focus of this paper is the *process* used to develop a successful empirical method. A six-stage process is described:

1. Identification of the problem, and of the end users of the final product
2. Development of a conceptual rock mechanics model, and identification of the key parameters in that model
3. Identification of measures for each of the key parameters, and the development of new measures (such as rating scales) where necessary
4. Data sources and data collection
5. Statistical analysis
6. Packaging of the final product

Each of these stages has its own potential rewards and pitfalls, which will be illustrated by incidents from the author's own experience. The ultimate goal of this paper is to provide a new and deeper appreciation for empirical techniques, as well as some guidelines and opportunities for future developers.

INTRODUCTION

Design is the central engineering activity. It is a process which combines knowledge and judgement to obtain a desired outcome. Models are a crucial element in the design process, even though all models are limited in their ability to represent real systems.

In their seminal 1988 paper, Starfield and Cundall introduced a classification of modeling problems (Figure 1). The X-axis measures the level of understanding of the fundamental mechanics of the problem to be solved. The Y-axis refers to the quality and/or quantity of the available data, including material properties, boundary conditions, and past experience. In many branches of mechanics, most problems fall into region III, where there is both good understanding and reliable data. This is the region where numerical models can be built, validated, and used with conviction.

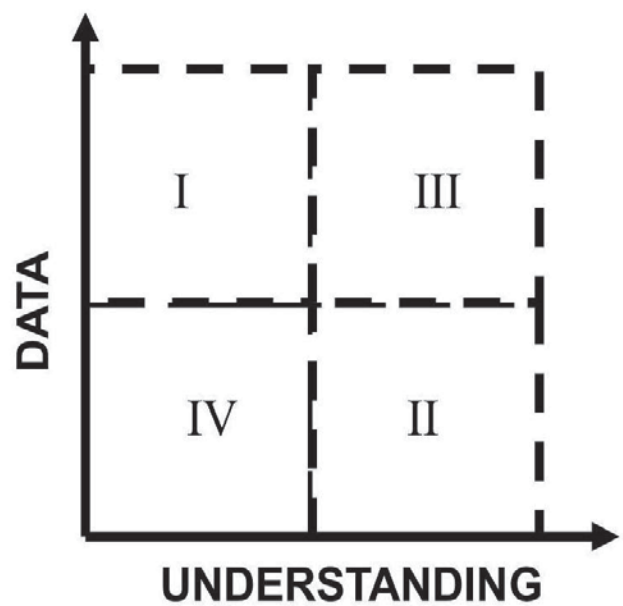

Figure 1. Classification of modeling problems (after Starfield and Cundall, 1988).

Starfield and Cundall argued that problems in rock mechanics usually fall into the data-limited categories II or IV. The "phase diagram" shown in Figure 2 helps explain why. It indicates that the three "end-members" of rock mass behavior are:

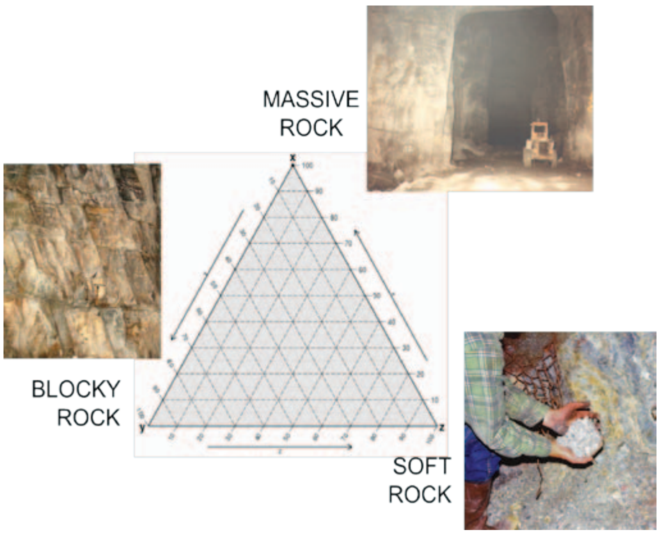

Figure 2. “Phase Diagram” for rock mass failure mechanisms in mining.

- Massive, strong rock that behaves elastically and is subject to brittle failure
- Blocky rock, where deformation and failure occurs exclusively along well defined joint systems
- Soil-like rock, which is subject to shear failure through the rock mass

Most real rock masses fall somewhere in the middle of this phase diagram. This is why it has proved so difficult to build and use numerical models. It is not enough that the model itself incorporates the many different failure modes, but it must have quality input properties and boundary conditions (in situ stresses) to match. Starfield and Cundall concluded that a more experimental use of models was appropriate for geomechanics.

In the field of mining ground control, however, many problems actually fall into Starfield and Cundall’s region I. Our understanding of the complex mechanical behavior and properties or rock masses may be limited, but the potential for data collection is huge. Hundreds of stopes and panels are mined each year, and each one is a full-scale test of a mine design. As Jack Parker noted in 1974, “Scattered around the world are millions and millions of pillars–the real thing–under all imaginable conditions; and tabulating their dimensions, the approximate loads, and whether they are stable or not would provide most useful guidelines for pillar design.”

Actually, simply tabulating data does not necessarily lead to useful conclusions. Fortunately, today’s data analysis techniques are far more powerful than those that were available to the mine design pioneers. In the past 30 years, sciences like economics, sociology, psychology, anthropology, and epidemiology have all been transformed by quantitative data analysis using statistics. Sophisticated statistical packages enable researchers in those fields and others to efficiently comb large databases for significant relationships between the variables. Even more recently, the business models of some of the most successful corporations in the world are based on “mining” the immense quantities of data available from internet searches, social networking, cell phone usage, and many other sources.

THE HISTORY OF EMPIRICAL DESIGN

For thousands of years, all mine design was empirical, in the sense of being based on past experience rather than engineering mechanics. However, the first empirical design method that combined case history data with rock mechanics principles appears to have been the one published by Bunting in 1911. Bunting addressed the issue of pillar sizing for the anthracite coalfields of eastern Pennsylvania. Improper pillar design had caused numerous squeezes, whose “inherent effects” were “the crushing of the pillars, the caving of the roof, and the heaving of the bottom.” After testing hundreds of coal specimens, Bunting concluded that the laboratory strength of anthracite could be represented as:

$$S_s = 1750 + 750\ (w/h)$$

Where:

S_s is the specimen strength (in psi),

w is the specimen width, and

h the specimen height.

Critically, however, Bunting also had full-scale data in the form of data from actual squeezes. He concluded that the laboratory specimens were approximately 2.5 times stronger than full-size pillars, such that the pillar strength (Sp) was:

$$S_p = 700 + 300\ (w/h)$$ [1]

Figure 3 shows Bunting’s data, and his design curve.

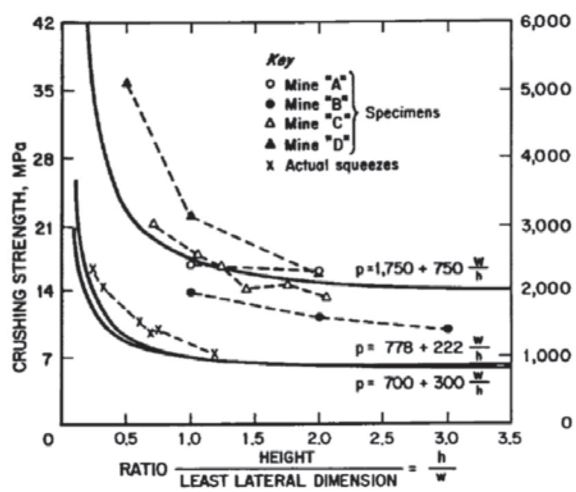

Figure 3. Empirical formula for the strength of anthracite pillars proposed by Bunting (1911).

Miklos Salamon was responsible for the next significant advance in the science of empirical design (Salamon and Munro, 1967). Following the infamous Coalbrook pillar collapse in which more

1 It may be noted that Bunting’s equation can be rewritten as Sp = 1000 (0.7 + 0.3 (w/h)), which may be compared to the square pillar form of the Mark-Bieniawski equation Sp=900 (0.64 + 0.36 (w/h)). It seems that a century of research has succeeded only in expanding the equation’s accuracy by one significant figure!

than 400 South African coal miners died, Salamon was asked to develop guidance to prevent a re-occurrence. First, he collected a case history database of 27 failed and 98 unfailed areas of room and pillar workings. Then he modeled the strength of the pillars using a simple power function, using just the pillar's width and height as input. The model contained three unknown constants, which were estimated using the "maximum likelihood" statistical technique. The resulting "Salamon-Munro formula," or some version of it, has been used in the design of nearly every pillar mined in South Africa since.

Looking back 20 years later, Salamon (1989) wrote that empirical methods were a "very powerful, and to an engineer, very satisfying technique to solve strata control problems…. The main advantage of this approach is its firm links to actual experience. Thus, if it is judiciously applied, it can hardly result in a totally wrong answer." Salamon did however caution that the developer of an empirical method must start with "a reasonably clear understanding of the physical phenomenon in question. This is a feature which distinguishes it from ordinary regression used in statistics."

The next major breakthrough was the development of modern rock mass classification systems in the early 1970's. Today it is hard to imagine the field of rock engineering without the Geomechanics Rock Mass Rating (RMR) and Rock Tunneling Quality (Q) systems. Rock mass classifications have been successful because they (Bieniawski, 1988):

- Provide a methodology for characterizing rock mass strength using simple measurements;

- Allow geologic information to be converted into quantitative engineering data;

- Enable better communication between geologists and engineers, and;

- Make it possible to compare ground control experiences between sites, even when the geologic conditions are very different.

The last point is a key reason why rock mass classifications play such an essential role in empirical design. By reducing the overwhelming variety of geologic variables into a single, meaningful, and repeatable parameter, they make it possible to quantify geology and include it in statistical analysis.

The original application of both the Q and RMR systems was to the selection of support for tunnels (Bieniawski, 1973; Barton et al., 1974). The Q system in particular was associated with a very large case history database, which underpinned a very detailed set of support recommendations. Because of their tunneling focus, both systems included parameters that went beyond geologic rock mass characterization. The Q system incorporated a factor addressing the in situ stress level, while the RMR evaluated the orientation of major discontinuities relative to the tunnel drivage. Both also included groundwater factors.

The rock mass classification concept was quickly transferred to mining. Laubscher (1977; 1990), and Jakubec and Laubscher (2000), developed a Modified Rock Mass Rating (MRMR) system which starts with the basic RMR value and adjusts it to reflect the changes introduced by mining activities. The four "adjustment factors" address weathering, mining induced stresses, joint orientation, and the effects of blasting. Originally, block caving mines in southern Africa provided the data for the MRMR and its design recommendations, but other case histories from around the world have been added since. A set of support recommendations was developed as well.

Kendorski et al (1983) also modified the RMR to produce the MBR (modified basic RMR) system, based on case histories collected from block caving operations in the USA. The MBR involved adjustments for blast damage, mining induced stresses, structural features, distance from the cave front, and the size of the caving block. Support recommendations were presented for isolated or development drifts as well as for the final support of intersections and drifts. Unfortunately, Kendorski's system was released just before many block caving mines in the US were permanently closed (Atchison, 1984), and so it was never widely adopted.

The Stability Graph method was first developed by Mathews et al. (1981), and was substantially extended by Potvin (1988) and Nickson (1992). By the mid 1990's its database included more than 350 stope stability case histories from Canadian underground mines (Hoek et al., 1995). A substantial international literature about the method is indicative of its worldwide application (Potvin, 2014).

The Stability Graph method begins with the Q rating, shorn of its groundwater and stress parameters. The modified Q' rating therefore focusses on the rock mass structure (joint distribution) and the joint conditions (joint shear strength). The Q' rating is then multiplied by "adjustment factors" representing the stress acting on the stope relative to the uniaxial compressive strength of the rock, and the orientation of the most significant discontinuities relative to the stope walls and back. The resulting "Stability Number" (N') is then plotted against the "Hydraulic Radius," which is a measure of the stope span and shape (Figure 4). Three zones are shown, together with "transition zones" between them:

- Stopes that are likely to be stable without support
- Stopes that are likely to be stable with cable bolt support, and
- Stopes that will likely suffer significant stability and dilution problems ("caving")

When using the Stability Graph method, a different N' is calculated for each stope wall and for the stope back, and then the designer can use those values to select both the stope dimensions and the cable bolt density.

In early versions of the method, the boundaries between these zones were apparently drawn by hand, based on the case history data (hence the name "Stability *Graph* Method.") Nickson (1992) used discriminant analysis to help define the zones shown in Figure 4, and also to develop guidelines for selecting cable bolt support density. Mawdesley et al. (2001) extended the database to very large stopes, and used logistic regression to help define the stability zones. Other important contributions to the technique have included estimates of the "equivalent linear overbreak/ slough" (ELOS) and stope design in very weak rock masses (Pakalnis, 2014).

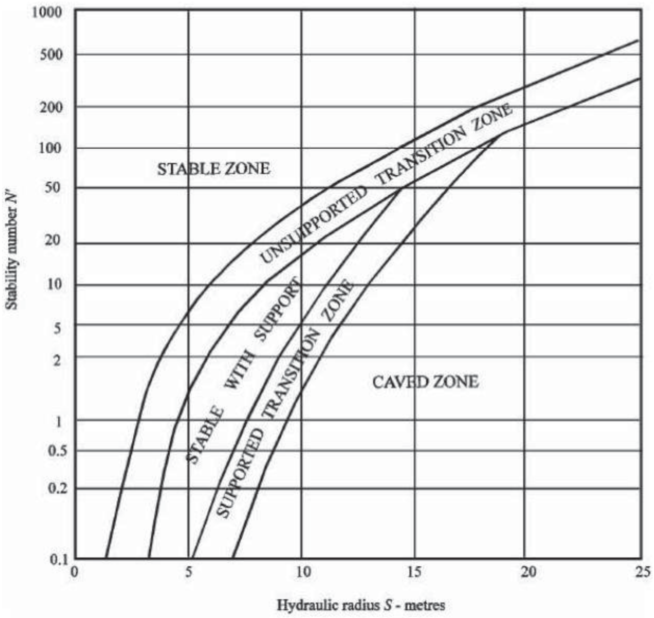

Figure 4. Zones defined by the Stability Graph method (after Nickson, 1992).

All these mining rock mass classification methods were originally developed before the advent of widely accessible statistics packages, so the lack of quantitative statistical analysis is understandable. Without the aid of statistics, their creators had to rely entirely on their own judgment to develop both the rating scales for each of the individual "adjustments," and the relative weights of all of the adjustments within each system. While the solid track record of the MRMR and the Stability Graph methods testify to the caliber of those judgments, in retrospect it is easy to see how they could have benefitted from more rigorous statistical analysis.

Pillar design for hard rock has been the subject of several empirical techniques, though none has achieved the widespread acceptance of the ones used for stope design. The pioneering work of Hedley and Grant (1972), who applied the Salamon-Munro approach to a database of 28 pillar designs from the Elliot Lake mining district, is still widely referenced. Lunder and Pakalnis (1997) proposed a "Confinement Formula" based on an analysis of seven combined databases with a total of 178 case histories. Most recently, Esterhuizen et al. (2011) developed the S-Pillar design guidelines for underground limestone mines. Their method is combined a comprehensive survey of 91 pillar layouts in 34 US stone mines with numerical analyses of the effects of large angular discontinuities, weak bands, and benching on pillar strength.

In coal mining, the first of the modern empirical techniques was the Analysis of Longwall Pillar Stability (ALPS) method (Mark, 1990; Mark et al., 1994). The need for ALPS was triggered by the rapid growth of longwall mining in the US during the 1980's, together with the Wilberg Mine Disaster which claimed 27 lives in 1984. A blocked tailgate entry was one of the causes of the great loss of life at Wilberg, and subsequent new regulations required that longwall tailgate entries be available for emergency egress at all times.

ALPS initially focused on extending existing pillar strength formulas with new equations for estimating retreat mining abutment loads. However, while many mines had found by trial and error that tailgate conditions could improve significantly when pillar sizes were increased, it was also evident that pillar design was not the only factor affecting tailgate *entry stability*. European studies from the 1960's had concluded that "whether or not the stress (from an extracted longwall panel) will influence a roadway depends more on the *rocks which surround the roadway itself* than on the width of the intervening pillar" (Carr and Wilson, 1982). Roadway stability might also be expected to be affected by roadway width and roof support.

To evaluate coal mine roof, the existing rock mass classifications were tried and found wanting, because:

- They tended to focus on the properties of *joints*, when *horizontal bedding* is generally the most significant discontinuity affecting coal mine roof.
- They rate just one rock unit at a time, while coal mine roof often consists of several layers that vary in strength.
- They apply to unsupported rock, while roof bolts are universally employed in coal mines, and so the "bolted interval" must be treated as a single structure.

The Coal Mine Roof Rating (CMRR) was developed to meet these requirements (Molinda and Mark, 1994; Mark and Molinda, 2005). It employed the familiar format of Bieniawki's RMR, summing the individual ratings to obtain a final CMRR on a zero to 100 scale. It was also calibrated so that the CMRR/standup time relationships are roughly comparable to the ones determined for the RMR. On the other hand, the specific input parameters and weightings within the CMRR were largely new and derived from the rich vein of experience with coal mine ground control going back to the 1970's (Mark and Molinda, 2005).

The data for the original CMRR was collected from underground exposures, primarily roof falls. The input parameters included the strength (UCS) and moisture sensitivity of the intact rock, and the shear strength, roughness, spacing, and persistence for the bedding and other discontinuities. After an early false start, a core-based CMRR was also developed. The three inputs it requires are the UCS, the RQD or fracture spacing observed in the core, and diametral Point Load Test (PLT) values as a measure of bedding strength.

The tailgate stability problem could now be attacked using the CMRR as the measure of rock quality (Mark et al., 1994). A database of case histories was collected from 44 longwall mines, and discriminant analysis was used develop guidelines for the ALPS Stability Factor (SF) based on the CMRR and other factors (Figure 5). A similar process was followed by Colwell et al. (1999) in Australia, and resulted in the Analysis of Longwall Tailgate Serviceability (ALTS).

The Analysis of Roof Bolt Systems (ARBS) was also made possible by the CMRR (Mark et al., 2001). ARBS is based on studies of roof fall rates conducted at 37 mines, and uses the number of roof falls per 10,000 feet of drivage as the outcome variable. ARBS provides recommendations for the primary support pattern, as well as a suggested bolt length. Statistical analysis

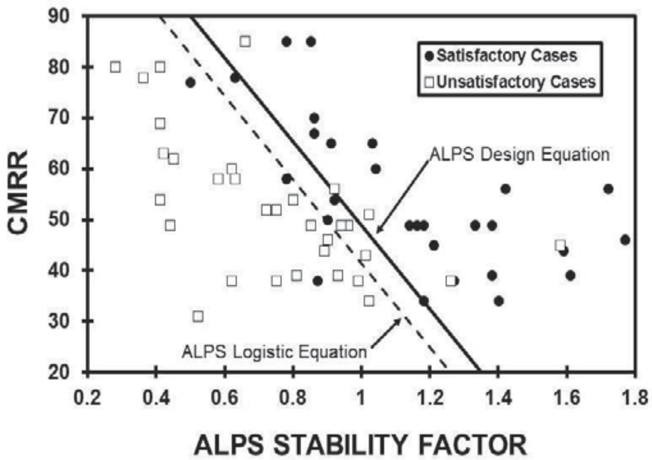

Figure 5. The ALPS database and logistic regression analysis (Mark et al, 1994).

indicated that in addition to the CMRR, the other significant factors were the span and the depth of cover.

Other empirical methods developed for coal mine entry stability using large databases of case histories include:

- Evaluation of where extended cuts (or "cut-and-flit" mining in Australia) may be suitable (Mark, 1999)
- Estimation of support requirements for longwall mining through pre-driven rooms (Oyler et al., 1998; Thomas, 2008)
- Design of extra-wide roadways for longwall set-ups and other applications (Thomas, 2010; Colwell and Frith, 2013)
- Analysis and Design of Rib Support (ADRS) for mains and gateroad entries (Colwell and Mark, 2005)

The most significant development in the area of coal pillar design has been the development of Analysis of Retreat Mining Pillar Stability (ARMPS) program. Just as ALPS is actually misnamed (since it addresses the stability of the longwall tailgate *entry*, not the *pillars* per se), the applicability of ARMPS also goes far beyond retreat mining. ARMPS was developed to prevent pillar squeezes, massive pillar collapses, and coal bursts. It employs the same "abutment angle" loading model as ALPS, extended to three dimensions. It can also model a wide variety of mining geometries, including both production pillars and barrier pillars.

The original ARMPS database included 140 room-and-pillar mining case histories (Mark and Chase, 1997). While the method explained the data very well up to depths of 650 feet or so, it seemed that the calculated SF decreased under deeper cover for both successful and unsuccessful cases. Two subsequent research projects added another 500 cases to the database, mainly from deep cover mines (Chase et al, 2002; Mark, 2010). The ARMPS loading model now incorporates a pressure arch function, whose form was determined through statistical analysis of the case history data.

The Analysis of Multiple Seam Stability (AMSS) method required the most sophisticated statistical analysis of any of the methods described here (Mark, 2007). Multiple seam interactions are very complicated phenomenon, and many factors can potentially be involved. The AMSS database included 344 case histories, each of which was defined by 22 variables. Logistic regression was used to winnow these down to six key parameters, which were then combined into a predictive equation. AMSS also can be used to predict both pillar failures (squeezes) and roof stability issues.

A HOW-TO GUIDE TO EMPIRICAL DESIGN

Empirical design can seem deceptively simple. What could be easier than collecting case histories, plotting them up, and drawing a line separating successes from failures? In reality, a *successful* empirical method, one that meets a real industry need and reliably provides safe and cost-effective design solutions, is the result of a complex development process. The process consists of six specific stages, each of which has its own potential rewards and pitfalls:

1. Identification of the problem, and of the end users of the final product
2. Development of a conceptual rock mechanics model, and identification of the key parameters in that model
3. Identification of measures for each of the key parameters, and the development of new measures (such as rating scales) where necessary
4. Data sources and data collection
5. Statistical analysis
6. Packaging of the final product

While the developers of the successful empirical methods described in the preceding section may not have been consciously aware that they were following this process, in retrospect it can be seen that they were. It should also be recognized that the stages in the process are not necessarily sequential, and in fact require feedback loops as learnings occur. For example, the statistical analysis (Stage 5) may result in a finding that requires a modification to the model (Stage 2).

Throughout the process, the developer attempts to balance *simplicity* and *accuracy.* A complicated method, or one that requires hard-to-obtain input parameters, will be difficult to develop and probably difficult to use. Moreover, since mining case history databases are relatively small compared with those used in other scientific fields, the number of parameters in the model must also be kept small. On the other hand, a model's ability to simulate a real world system depends upon its incorporating all of the important characteristics of the real thing.

One of the great strengths of empirical methods is that they can largely side-step the need for accurate rock mass properties that bedevils numerical techniques. They can do this because the analysis essentially involves *comparing* case histories to one another. Accuracy is only required in a *relative* sense, not an *absolute* one. In other words, it is not necessary to know the true strength of the pillars in Case A, it is sufficient to know that they are stronger than those in Case B and less strong than those in Case C.

Stage 1: Identification of the Problem

The essential initial step is a clear definition of the problem being addressed. A muddled understanding of the problem can cascade through the entire process and result in complete confusion. In particular, it is critical that the *failure mode* be clearly identified. To give a simple example, if case histories involving floor failure are mixed together with actual pillar failures, then it may be impossible to learn anything about either one.

Multiple seam mining provides another illustration of this point. Multiple seam interactions baffled researchers for decades, both in the U.S. and internationally. For example, one group of researchers found that "stresses from superincumbent workings are not transferred through shale strata for distances of over 110 ft" (Haycocks et al., 1982), while another group indicated that "a [vertical] stress transfer distance of 760 ft has been recorded between longwalls" (Haycocks et al., 1992). Clearly, no amount of statistics could resolve such discrepancies. As it turns out, however, multiple seam interactions are caused by four distinct types of failure (Mark, 2007):

- *Undermining*, where full extraction was previously conducted in an overlying seam
- *Overmining,* where full extraction was previously conducted in an underlying seam
- *Ultraclose*, where only first workings are present in the previously-mined seam, and
- *Dynamic*, when active mining occurs above or beneath open entries that are in use.

Each of these mechanisms, and particularly dynamic interactions involving full extraction beneath active workings, result in interaction distances that are significantly different from the others. Once the problem was understood correctly, it was possible to single out the first two for solution with AMSS (Mark et al., 2007). An empirical method must be clear about which types of failure it addresses, and which ones are beyond its scope.

Stage 2: The Conceptual Rock Mechanics Model

The empirical approach requires that the researcher begin with a clear hypothesis, in the form of a simplified model of the real world that abstracts and isolates the factors that are deemed to be important to the problem at hand. For example, an entry or stope stability model will normally need to include:

- The rock mass quality
- The applied stress
- The orientation of discontinuities relative to the mine opening
- The size and shape of the opening
- The characteristics of installed support
- An outcome variable defining success or failure of the design

A general pillar stability model includes these main elements:

- The strength of ore or coal
- The pillar geometry (w/h ratio)
- Discontinuities within the pillar
- The roof and floor strengths
- Pillar loading (including those due to development, retreat mining, multiple seam interactions, etc)
- An outcome variable defining success or failure of the design

The number of elements included in the model may be winnowed down based on the current scientific understanding. For example, studies have shown that variations in laboratory coal strength normally have minimal effect on the strength of full-scale coal pillars, and coal pillars are normally so squat in shape that discontinuities have minimal effect on strength. So these two factors are usually ignored in coal pillar design, but both were included in the S-Pillar method developed for the design of slender limestone pillars (Esterhuizen, 2011).

On the other hand, each of these main elements is very complex, and may in turn be determined by a number of variables. The stress applied to an opening, for example, is a function of the far-field tectonic stress field, the depth of cover, the orientation of the mine opening, its size and shape, and the stiffness of the rock surrounding it. Whether all of this can be represented in the model by a single parameter, or whether several parameters may need to be included in the model, depends upon the problem being analyzed and the data that is available.

It might be tempting to simply create a long list of variables that might impact the result, and then "let the statistics sort it out." This is unlikely to be a successful strategy, because the number of variables quickly becomes too great for the size of the case history database. Therefore prudent simplification is necessary. Moreover, many of the variables *interact* with one another. For example, the magnitude of an abutment load depends on both the depth of cover and the extent of the mined out area. Those two variables must be combined in some fashion for the statistical analysis to have a chance discerning the effect of either one. The modeler must use both their knowledge of the science and their engineering judgment in making these decisions.

It is essential, however, that no important parameter be left out simply because it is difficult to measure. To provide one example, individual stress measurements were not available during the development of the Extended Cut database. Without a parameter representing the stress, the statistical significance of the early analysis relating entry width and CMRR to stability was very low. Once the depth of cover was included in the model, however, the statistical significance improved greatly. Evidently, the depth of cover was an adequate *surrogate* or *proxy* for horizontal stress. Later research confirmed that the horizontal stress in the coalfields is strongly correlated with the depth of cover (Mark and Gadde, 2008). But the important point is that an imperfect measure is far superior to none at all.

Stage 3: Parameter Development

Parameter development is closely linked to the process of model development described above. A number of options are available to the modeler:

- Direct measurements
- Rating scales
- Simple mechanics-based models
- Numerical models

Parameters that can be measured directly include the depth of cover, the UCS, and the opening dimensions. The developer must also decide the most appropriate forms of these parameters. For example, as measures of the opening dimensions, ARBS uses the sum of the intersection diagonals, while the Stability Graph method uses the hydraulic radius. In fact, at this early stage, it is appropriate to employ several different parameters that measure the same element in the model. Later on the statistical analysis can determine which one is the best predictor.

Rating scales make it possible to include critical variables that are difficult to measure directly. Rock mass classifications are the best-known examples of rating scales used in empirical techniques. In fact, "rock mass classification" and "empirical technique" are sometimes considered synonymous, which is definitely not the case. A rock mass classification is best considered as single number which summarizes the key geotechnical characteristics of the rock mass. In this writer's opinion, it is better to keep other "adjustment factors" (like stress level) separate from the rock mass classification. For one thing, keeping the adjustment factors separate allows the statistical analysis to help determine the relative weights of each one in the final design. In addition, when all the factors are combined with the rock mass rating into a single number, it is harder for users to understand which parameters are the most critical in any particular application.

The development of a good rating scale typically requires a significant amount of experience and engineering judgment. For example, Laubscher's "blast damage adjustment" was not based on any explicit measurements or models, but rather on "numerous observations in the field" (Laubscher, 1990).

Rating scales must also be simple enough that users will be able to understand them and to collect the necessary data. For example, remnant pillars left in old works are the major cause of multiple seam interactions, and they come in a wide variety of configurations. A complicated rating scale that tried to separate the types of remnants into too many classes might have proved unworkable, and might not have improved the reliability of the model in any case. Ultimately, the remnant pillar variable developed for AMSS has just two levels: (1) Gob-solid boundary, or (2) Isolated remnant pillar.

Rating scales have also been used to evaluate the density of roof support. For example, in ARBS, the parameter that is used to measure the roof bolt density is:

$$ARBS = \frac{(L_b)\,(N_b)\,(C)}{(S_b)\,(W_e)}$$

Where: L_b= Length of the bolt (ft)

N_b = Number of bolts per row,

C = Bolt capacity (1000's of pounds, (or kips)) per bolt

S_b = Spacing between rows of bolts (ft),

W_e = Entry width (ft)

This parameter has been criticized because its "units," which work out to be "kips per ft," is not really a measure of support capacity. However, most of the rating scales used in empirical models are entirely unitless, and so a parameter like ARBS is also best understood as a unitless rating scale. Again, because the statistical analysis really consists of relative comparisons between case histories, even a physical measurement like span or UCS is just a rating scale in the model. There will be more to say about this during the discussion of statistics.

Another technique for developing a single parameter to represent a complex physical phenomenon is to create a simple, mechanics-based model, using the available scientific knowledge and data as appropriate. The abutment angle concept used in ALPS and its cousins is a good example of this process. It built on previous work by Peng, Wilson, and Choi, and was calibrated using actual stress measurements (see Mark, 1990 for details). While its estimates of the actual stress that might be measured in real pillars may not be highly accurate in all cases, it does capture the key elements of the process of abutment load formation and so has served its purpose well. The pillar strength formulas used in ALPS and other pillar design methods are also examples of model parameters based on simple mechanics-based models.

Numerical models can be used to generate model parameters in a similar fashion. Hoek et al. (1995) show how a simple, two-dimensional elastic model can be used to obtain the stress term for use in the Stability Graph method. AMSS employs a built-in, two-dimensional version of LaModel to estimate multiple seam loads (Mark et al., 2007). The S-Pillar method incorporates several rating scales that were developed through numerical modeling (Esterhuizen et al., 2011).

So far this discussion has addressed the *predictor* parameters that are included in the model. The single most important parameter, however, is the *outcome*. It is essential that the outcomes be clearly defined, to ensure that every case history is properly categorized. Every success must clearly have been successful, and every failure the result of the mechanism(s) being modeled. Miscategorizing even a few successes as failures, or vice versa, can entirely confuse the statistical trends in the data. A database that unwittingly mixes failure mechanisms may also be compromised.

An objective measure of the outcome, one that does not rely solely on the subjective opinions of different observers, is also highly desirable. For example, in the ARBS database, the outcome variable was the number of MSHA-reportable roof falls per 10,000 ft of drivage (Mark et al., 2001). As discussed in the next section, however, it is not always possible to employ such an objective measure.

Stage 4: Data Collection

The case history database is the true heart of any empirical method. Empirical methods use past experience to guide future design, and the database is where the past experience resides.

Sufficient case history data is seldom available from the literature or other published sources. It is also unlikely that many failures will be available for first-hand observation. Failures are typically rare events that are likely to have occurred in the distant past in locations that are no longer accessible. Therefore reliable sources at a number of mines, particularly local experts who know the history, are essential. If those individuals also have a basic understanding of ground control, so much the better.

It is the author's experience that the best data is often obtained while looking at mine maps, and going over the mine's experience on a panel-by-panel basis. A map may show, for example, that some pillars were abandoned in a certain panel. Perhaps this was due to local pillar failure, but the pillars might also have been left because of seam dips, low coal heights, low ore grade, excessive groundwater, or some other issue unrelated to ground control.

A key pitfall at the data collection stage is to "collect information to the model," rather than attempting to fully document the conditions and experience at the mine. While detailed questionnaires are essential to ensure that the necessary minimum information is collected, the discussions at the mine should be allowed to range much further. For one thing, the research may lead the project in unexpected directions, and the initial parameters may be modified or new ones may be added. It is seldom possible to return to the all the sources in order to populate a new parameter with data, but that might not be necessary if detailed evaluations were done. The principle should be "Better too much data than not enough!" This principle applies also to core logs, rock mechanics test data, and other documentation that might be available.

A second reason to conduct a complete evaluation is that some case histories are likely to be statistical outliers. The information collected at the mines may provide clues that can help explain why. There may be a unique factor—such as a major geologic feature—that is not present in the other cases. Sometimes an outlier is the clue that leads to a better understanding of the problem and a more refined model. On the other hand, an outlier may just represent the tail of an apparently random probability distribution.

A healthy skepticism regarding local explanations about the causes of events is generally in order. As Stemple (1956) noted while he was collecting his multiple seam database:

"Bad roof conditions are present in many cases where there is no vertically adjacent mining, and so it is not always possible to state definitely that mining in another seam is responsible for the conditions observed. *The coal miner is usually anxious to fix the cause of any difficulty which arises, and certainly the previous mining of a contiguous seam provides a convenient scapegoat*."

In addition, a "success" at one mine might be considered a "failure" at another, depending on the past experience and safety culture at each. Therefore it is essential that some more objective measures of the outcome be employed. For example, a "success" might be defined as "zero ground control-related delays or additional support," while a failure might mean "the map shows that the panel was abandoned prematurely, and sources at the mine confirm that the cause was poor ground conditions." However, even case histories with intermediate or undetermined conditions represent experience that should be retained in the database. While those cases may be excluded from some statistical analyses, they could also be very helpful in others.

While it is rarely possible to access more than a few of the historic case history sites, underground investigations should still be a part of the data collection process wherever possible. Underground observations provide a sample of the ground conditions and support performance associated with the mine, and they can also provide raw data on roof geology and strength for rock mass classification.

Stage 5: Statistical Analysis

The very words "statistical analysis" seem foreign to many in rock engineering. Engineers are trained to see the world in terms of load and deformation, where failure is simply a matter of stress exceeding strength. Statistics are generally given short shrift in engineering curriculums, and so the entire language of statistics is unfamiliar.

Yet statistics are the tools that science has developed to deal with uncertainty and probability. There are plenty of both in mining ground control! In fact, simply "eyeballing" a line through a scatter of data on a graph is a pseudo-statistical analysis that provides a sense of the trend of the data and the strength of that trend. The Stability Graph method apparently began life just this way.

The key point is that *statistical analysis assists engineering judgment; it does not substitute for it*. Hard and fast rules about when an r-squared is "statistically significant" are appropriate for process engineering, but they do not apply here. Statistics help interpret large data sets, but ultimately it is still the engineering judgment that counts.

If all rock engineering problems involved just two variables, x-y graphs and histograms might be all the statistics that were needed. In general, however, there are several variables involved. One solution is to combine several variables into one, thereby turning a multivariate problem into a bivariate one. But combining variables pre-supposes the relationships between them, which places a large burden on the judgment of the model developer. Multivariate statistical techniques are an alternative.

The most common multivariate technique is multiple regression, which takes the form:

$$Y = B_0 + B_1x_1 + B_2x_2 + \ldots + B_nx_n$$

Where Y represents the outcome, each x represents an input parameter, and each B represents a coefficient (slope) for its associated parameter. The coefficients (B's) are estimated by the regression analysis. In essence, the coefficients reflect the relative importance of each parameter to the outcome. Parameters with low statistical significance can be dropped from the model. The software also provides information regarding the statistical significance of the overall model, which can be used to compare models to one another.

Traditional multiple regression requires that the outcome variable be linear and continuous (like "stability factor" or "stress"). When the outcome variable is binary (i.e., there are two possible outcomes, as in "success" and "failure"), then logistic regression is the most common multivariate statistical technique. Logistic regression has much in common with linear regression. In both cases, the goal is to predict the outcome as a linear combination of the predictive variables. But in place of solving for "Y," logistic regression essentially solves for the likelihood ("Z") of a particular case having one of the two possible outcomes.

Figure 5 illustrates how logistic regression was used to develop the ALPS/CMRR design method. The logistic equation obtained from the statistical analysis was:

$$Z = 4.1\ (\text{ALPS SF}) + 0.057\ (\text{CMRR}) - 6.33$$

Where Z=0 means an equal probability of success or failure. This equation can be re-arranged to solve for the ALPS SF in terms of the CMRR:

$$\text{ALPS SF} = 1.67 - 0.014\ \text{CMRR}$$

This equation is shown on Figure 5 as the "Logistic Equation." The more conservative Design Equation that was adopted is also shown.

The process of model-building with logistic regression is also similar to that with linear regression. In general, the goal is to obtain a parsimonious model that best explains the data with the fewest variables. The statistical analysis is typically an iterative process. Many variations on the basic model can be tested and evaluated. As more refined models are developed with new combinations of variables, it is often necessary to test parameters that had previously been excluded to ensure that they were still non-significant.

Another important issue is that of correlations between variables within the database. In general, when two variables are highly correlated with one another, both cannot be used in the model. In the ALPS database, for example, the CMRR was found to be highly correlated with two other parameters, the primary support and the entry width. It appeared that where the roof was weak to start with, the mines had responded by installing more roof bolts and by narrowing their entries. As a result, of the three parameters, only the CMRR was included in the final model. However, the data was used to provide guidelines regarding what primary support and entry width should be used based on the site-specific CMRR.

It is also important that any continuous variables used in the model have a linear effect on the outcome. However, this is not a serious restriction, because it is possible to "transform" a variable so that its *effect* is linear. For instance, a 10-point decrease in the CMRR from 40 to 30 usually has a much greater effect on stability than a 10-point decrease from 80 to 70. If the effect of the CMRR is not linear, then using the log of the CMRR may be a better way to capture the effect of the CMRR in the regression, as was done during the development of both ARBS and AMSS. Transforming a variable in this manner is actually no different than plotting it on log paper.

One disadvantage of logistic regression is that there is no universally accepted measure of the overall model goodness-of-fit like the r-squared used in linear regression. However, there are a number of alternatives, such as the ROC, (or Receiver Operating Characteristic). Moreover, the user can select their own "optimal" cut point, where the likelihood of misclassifying a failure is considered acceptable. Finally, diagnostics are available that allow identification of outliers or individual case histories that have a great deal of influence on the model (Hosmer and Lemeshow, 2000).

Stage 6: Packaging the Final Product

If the first five stages have not been conducted properly, an attractive final package will not be able to make up for the lack of substance. On the other hand, a poor final package can ensure that an otherwise excellent and valuable empirical method languishes in obscurity.

The first, essential attribute of the method has to be that it provides reasonable, reliable, and useful guidance for engineering design. Fortunately, this is almost built into a properly constructed empirical technique. One potential pitfall is not making clear the limitations to the method. Empirical techniques are most reliable when they are interpolating within the boundaries of their case history database. It should be clear to users when a design problem falls outside those bounds.

A related requirement is that the users can understand the principles underlying the method, and the research that went it into its development. Again, a key advantage of empirical techniques is that they can normally be easily grasped by most mining professionals. However, the burden is on the developer to explain the system in clear but simple terms.

Clear procedures for input data collection are also essential. The simpler the input data, the better. In some cases, it may be useful to suggest alternative sources for particular pieces of input data. For example, UCS data may be obtained from laboratory tests, axial point load tests, ball peen hammer tests, or sonic velocity logs. Rock mass classification data is perhaps the most difficult type of input to expect others to collect in a uniform, objective, and repeatable manner. Several of the NIOSH software packages start with a low "default" value of the CMRR, which requires the user to justify higher values.

Today, a computer program, or at least a spreadsheet, is almost required if an empirical technique is to be widely accepted. Older tools, including written equations, graphs, and nomograms, are all historical artifacts. Software has many other advantages that go well beyond its ease of use. It is possible, for instance, for users to evaluate numerous "what-if" scenarios in a very short time. Such pseudo-parametric analyses should lead to better and more optimal designs. From the developer's standpoint, a big advantage is the ability to incorporate pop-up "warnings" when the use may be improper, such as when the input data is outside the range of the database. It is also possible to include extensive help files, tutorials, and reference materials.

EMPIRICAL DESIGN, LOCAL CALIBRATION, AND "TEST AREAS"

Years ago, empirical methods were understood as being most applicable to feasibility studies. It was assumed that when site-specific guidelines could be developed from local historical data, they would be more reliable than generic guidelines (Mark, 1990).

Decades of experience has led to different conclusions, however. In general, it appears that greater weight should be given to the generic guidelines rather than to local calibration. There are at least four reasons for this:

1. The generic guidelines were developed from very large databases encompassing wide varieties of ground conditions, and any particular local conditions are probably covered.
2. As Potvin (2014) noted with regard to the Stability Graph method, the generic guidelines "have the clear advantage of being *intrinsically calibrated* through the thousands of case studies to which they have been applied during the past 30 years."

3. A local calibration based on a handful of cases might not include failures simply because failures are unlikely events. If the underlying likelihood of a failure is one-in-ten, then five local successes might give a false sense of security.
4. The generic database was collected in a uniform and consistent manner, particularly as regards the definitions of "success" and "failure." At the Crandall Canyon mine, a faulty local calibration of ARMPS contributed to the collapse that resulted in nine fatalities in 2007. Historic case histories at Crandall Canyon where conditions were "borderline" at best were judged "successes, and used to justify designs with even lower ARMPS SF values (MSHA, 2008).

"Test Areas" or "Experimental Panels" are really just variants on the "local calibration" theme. As Galvin (2010) observed, "there is a history in the coal mining industry of pillar collapses arising from the adoption of mining layouts that were first trialed in a so-called 'experimental panel'." Galvin was thinking mainly of the Coalbrook Mine disaster of 1960, but the collapse of the Retsof salt mine in 1994 (Scovazzo, 1997) also comes to mind. In both of these instances, narrow experimental panels consisting of small or "yield" pillars were judged to be successful. Subsequent, much wider "full scale" panels collapsed, and both mines were destroyed. In retrospect it is clear that the small pillars in the experimental panels were shielded from the full tributary area load by "pressure arches" that formed in the overlying strata. When the pressure arches broke down above the wider panels, the pillars were subjected to much higher loads than they had experienced in the experimental panels.

Indeed, the test panel philosophy is out of step with modern risk management principles. Surely the odds of failure need to be small, certainly less than one-in-ten, even where the consequences of failure are very low. Yet if the actual odds are only one-in-two, a test panel still has a 50-50 chance of resulting in a (misleading) positive outcome. That the result of a first coin flip is heads doesn't guarantee that every succeeding coin flip will also be heads.

THE FUTURE OF EMPIRICAL DESIGN

Today empirical methods are an integral part of mine design around the world. Just last year the First International Conference on Mine Design Using Empirical Methods was held in Lima, Peru, focused exclusively on hard rock mining. In coal mining, empirical methods are involved in the design of nearly every pillar developed in South Africa, the US, and Australia.

But what does the future hold? Has saturation been reached? Will numerical modeling finally make empirical methods obsolete? Or will new sources of data open up even greater possibilities?

As this paper has shown, the best-known and widely-used empirical techniques have addressed industry-wide issues like pillar sizing or stope design. Most of these are now decades old, and while valuable updates, adjustments and modifications continue to be developed, it seems unlikely that the established techniques will be supplanted by entirely new ones any time soon. They have demonstrated their effectiveness, and have been "intrinsically calibrated" through wide use, so there seems little incentive to start over. In addition, failures are much less common nowadays, though that need not be a major hurdle because the definitions can always be shifted. Longwall tailgate failures were rare in Australia even during the 1990's, so Colwell et al (1999) included borderline tailgates and those requiring standing support in their "modified unsatisfactory" category.

Site-specific numerical modeling is also unlikely to replace empirical techniques in the immediate future. While very exciting progress has been achieved in the development of software that can represent the large range of rock mass behaviors and failure modes that occur underground, the proportionally large number of necessary input properties is seldom available for mining applications. The underground measurements needed to validate the models, including ground deformations, stress changes, and support loads, are similarly scarce. Even during the recent "Super Cycle" there was little progress on this front, and it would be irrational to expect more in today's economic environment.

On the other hand, there is reason to believe that an entirely new era of empirical methods for mining may be dawning. We are living in the age of Big Data, where the cost of collecting and analyzing enormous databases is constantly decreasing. Mining geomechanics is certainly not leading the trend, but it has not been entirely left out either. For example, software is available that reads longwall leg pressures and shearer position every 20 to 30 seconds and creates an independent database of these values in a compressed format that allows fast access over networks. Various parameters can then be calculated for each leg during each set-release cycle or during each shear (see Figure 6). These data have been used to develop indicators that routinely give geotechnical engineers real-time warnings of developing conditions, such as significant weighting and the formation of roof cavities. A majority of Australian longwalls have already implemented the system (Trueman et al., 2011; Hoyer, 2012).

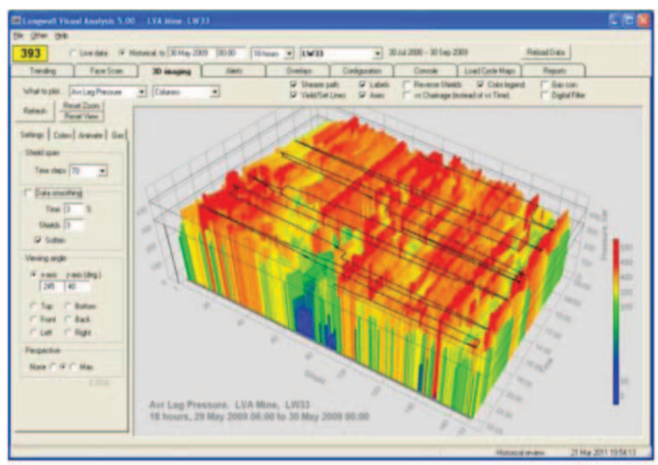

Figure 6. Presentation of data for early warning of longwall roof cavities using LVA software (Hoyer, 2011).

Extensometers, particularly two-point "tell-tales," have now been used routinely in many coal mines since the 1990's. Some mines have developed databases representing thousands of measurement locations, creating a valuable opportunity for back analysis. For example, the Springvale Mine in NSW, Australia, has related the observed deformation histories to a range of geotechnical factors, including the depth, the presence of faulting and other geological structures, roof lithology, secondary support timing, and the approach of the longwall face (Corbett et al., 2014).

The goal is to predict which areas are likely to eventually require supplemental support based on the mine's Trigger Action Response Plan (TARP). There are still challenges involved with the installation, monitoring, and interpretation of tell-tales, however. Remote monitoring is already available, however (Bigby et al., 2010), and surely this will only improve as wireless communication underground becomes routine.

Obtaining the necessary geologic and geotechnical data for these types of empirical databases remains a challenge. Considerable progress has recently been made towards interpreting geophysical logs for these purposes (Lawrence et al., 2013; Hatherly et al., 2013). Ultimately, it should be possible to employ data collected underground production drills for geotechnical characterization (Peng et al., 2005), but there appears to have been little progress in that direction lately.

Mine-wide seismic monitoring systems are also generating enormous amounts of data. Hudyma and Potvin (2010) report that the number of systems installed in deep Australian hard rock mines increased from one to thirty between 1995 and 2007. These systems can be used to locate seismic sources such as faults, dykes, shear zones, or geologic contacts, and to quantify the seismic hazard in terms of the largest seismic event that might be expected from a source. In addition, by relating the incoming peak particle velocities (ppv) to the actual damage experienced by installed support systems, it is possible to estimate the required support loads and energy dissipation requirements in dynamic environments (Mikula, 2012).

Another source of data is numerical modeling itself. Today's computing power makes it is possible to run many sophisticated, non-linear, three-dimensional models in a reasonable amount of time. Thus large databases of parametric analyses or pseudo case histories can be created, and treated as ersatz experimental data (as suggested by Starfield and Cundall (1988)!). These results can also be compared to existing empirical formulas (Esterhuizen et al., 2014). Numerical models can also be used to re-analyze empirical data bases, as has been done using LaModel with the ARMPS data base (Zhang et al., 2014). Alternatively, numerical modeling can be used to help develop rating scales that serve as elements in empirical models, as was done with S-Pillar (Esterhuizen et al., 2011).

In conclusion, the challenge for the future will be to turn these new sources of data into practical design tools. The author hopes that the basic principles outlined in this paper will be helpful in developing the next generation of empirical methods.

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